

Algebra 2

2.2: Intro to Quadratic Graphs

Objectives: Define, identify, and graph simple quadratic functions in standard form. Find maximum and minimum values.

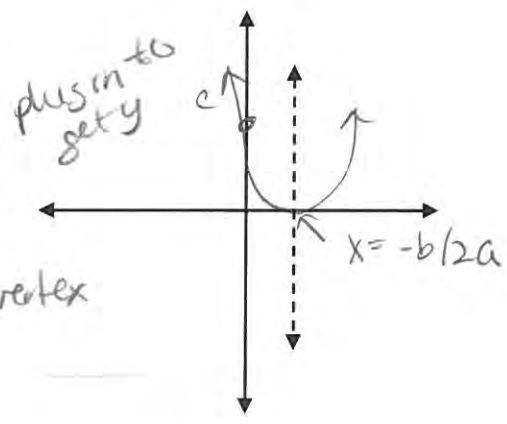
A Quadratic is a function that can be written in the form  $y = ax^2 + bx + c$  where  $a, b,$  and  $c$  are real numbers and  $a \neq 0$ . Ex:  $y = 5x^2$   $y = -2x^2 + 7$   $y = x^2 - x - 3$

Quadratic Function in standard form:  $f(x) = ax^2 + bx + c$   $a \neq 0$

$a =$ w/ $x^2$ tells $\uparrow$ or $\downarrow$ Quadratic term $b =$ w/ $x$ Linear term $c =$ constant by itself
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Graph of a quadratic function is a parabola with:

- y-int = (0, c)
- axis of symmetry  $x = -b/2a$
- x-coordinate of the vertex  $-b/2a$
- coordinates of the vertex  $(-b/2a, f(-b/2a))$



Domain:  $(-\infty, \infty)$   
 Range:  $[k, \infty)$  or  $(-\infty, k]$   $k = y$  of vertex

Example 1: Find axis of symmetry and vertex for

$f(x) = x^2 - 6x + 10$

$a = 1$   $b = -6$   $c = 10$

$x = -b/2a = 6/2(1) = 3$   $v(3, 1)$

$f(3) = 9 - 18 + 10 = 1$   $x = 3$  axis

You try:  $f(x) = x^2 - 8x + 5$

$a = 1$   $b = -8$   $c = 5$

$x = -b/2a = 8/2(1) = 4$

$f(4) = 16 - 32 + 5 = -16 + 5 = -11$

$v: (4, -11)$

$x = 4$  axis

Example 2. Graph.  $y = x^2 - 4x + 8$

Identify a, b, c,  $\underline{1, -4, 8}$

Find the axis of symmetry  $\underline{\frac{-b}{2a} = 2 \quad x = 2}$

Find the vertex  $\underline{f(2) = 4 - 8 + 8 = 4 \quad (2, 4)}$

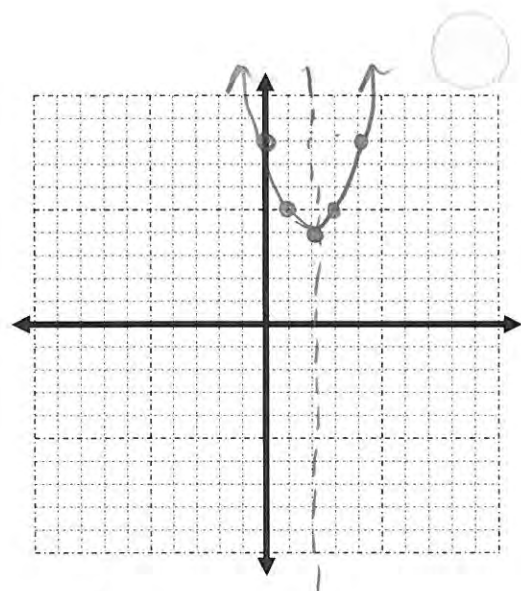
Find the y-intercept  $\underline{0 - 0 + 8 \quad (0, 8)}$

Use symmetry to plot another point.

Ask yourself, will the parabola open up or down?  $\underline{\uparrow}$   
 ○ Is the vertex a max or min?  $\underline{\text{min}}$

Make a table with numbers above and below the vertex if needed.

Domain:  $(-\infty, \infty)$       Range:  $[4, \infty)$



x	y
1	$1 - 4 + 8 = 5$
3	5

You try.

$f(x) = x^2 - 6x - 1$

Identify a, b, c,  $\underline{a=1 \quad b=-6 \quad c=-1}$

Find the axis of symmetry  $\underline{\frac{b}{2} = 3 \quad x = 3}$

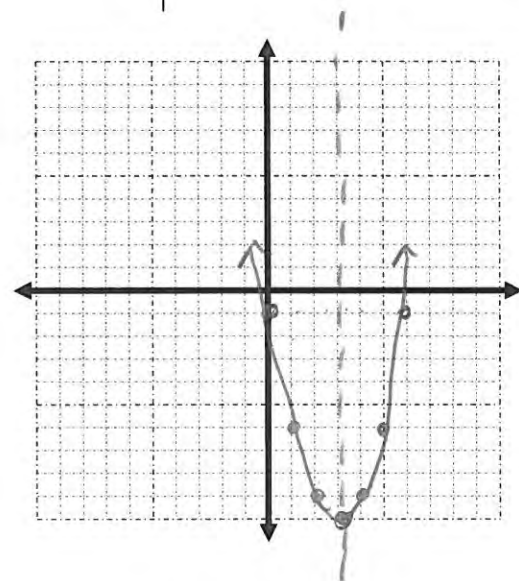
Find the vertex  $\underline{f(3) = 9 - 18 - 1 = -10 \quad (3, -10)}$

Find the y-intercept  $\underline{(0, -1)}$

Ask yourself, will the parabola open up or down?  $\underline{\uparrow}$   
 ○ Is the vertex a max or min?  $\underline{\text{min}}$

Make a table with numbers above and below the vertex.

Domain:  $(-\infty, \infty)$       Range:  $[-10, \infty)$



x	y
1	$1 - 6 - 1 = -6$
2	$4 - 12 - 1 = -9$

**Example 3:** The graph of the function  $f(x) = -10x^2 + 700x - 6000$  shows the profit a company earns or selling headphones at different prices. What is the maximum profit the company can expect to earn?

Find vertex

$$\frac{-700}{2(-10)} = 35$$

$$f(35) = -10(35)^2 + 700(35) - 6000 = 6250 \$$$

**You try.** A water balloon was thrown from a window. The height of the water balloon over time can be modeled by the function  $f(x) = -16x^2 + 160x + 50$ , where  $f(x)$  is the height in feet and  $x$  in the time in seconds. What was the maximum height of the water balloon after it was thrown?

vertex

$$\frac{-160}{2(-32)} = 5$$

$$f(5) = -16(5)^2 + 160(5) + 50 = 450 \text{ ft.}$$

**You try:** A local nursery sells a large number of ornamental trees every year. The owners have determined the cost per tree  $C$  for buying and caring for each tree before it is sold is  $C = 0.001n^2 - 0.3n + 50$ . In this function,  $C$  is the cost per tree in dollars and  $n$  is the number of trees in stock.

- How many trees will minimize the cost per tree? vertex
- What will the minimum cost per tree be?

$$\frac{+0.3}{2(0.001)} = 150$$

$$f(300) = .001(150)^2 - .3(150) + 50 = 27.5$$

**Example 4.** What is the equation of a parabola that passes through the points  $(-2, 32)$ ,  $(1, 5)$  and  $(3, 17)$ ?

$(-2, 32)$

$(1, 5)$

$(3, 17)$

$$32 = a(-2)^2 + b(-2) + c$$

$$5 = a(1)^2 + b(1) + c$$

$$17 = a(3)^2 + b(3) + c$$

$$32 = 4a - 2b + c$$

$$5 = a + b + c$$

$$17 = 9a + 3b + c$$

use  $y = ax^2 + bx + c$

$$y = 4x^2 + 3x - 2$$

$$5 = a + b + c$$

$$5 = 3 + 4 + c$$

$$-2 = c$$

$$32 = 4a - 2b + c$$

$$-5 = a + b + c$$

$$27 = 3a - 3b$$

$$135 = 15a - 15b$$

$$-45 = -15a - 15b$$

$$90 = 30b$$

$$3 = b$$

$$32 = 4a - 2b + c$$

$$-17 = -9a - 3b - c$$

$$15 = -5a - 5b$$

$$27 = 3a - 9$$

$$36 = 3a$$

$$4 = a$$

Do example 5 if time.

